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Brane-antibrane systems and the thermal life of neutral black holes

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Abstract

A brane-antibrane model for the entropy of neutral black branes is developed, following on from the work of Danielsson, Guijosa and Kruczenski [1]. The model involves equal numbers of Dp-branes and anti-Dp-branes, and arbitrary angular momenta, and covers the cases $p = 0, 1, 2, 3, 4$. The thermodynamic entropy is reproduced by the strongly coupled field theory, up to a power of two. The strong-coupling physics of the $p = 0$ case is further developed numerically, using techniques of Kabat, Lifschytz et al. [2,3], in the context of a toy model containing the tachyon and the bosonic degrees of freedom of the D0-brane and anti-D0-brane quantum mechanics. Preliminary numerical results show that strong-coupling finite-temperature stabilization of the tachyon is possible, in this context.

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1 Introduction

The drive to explain the thermodynamic entropy of black holes and black branes via the statistical mechanics of microscopic degrees of freedom has been a central preoccupation of string theorists and other gravitational theorists for at least two and a half decades. String theory has provided significant progress on this front in the last eight years. The celebrated 1996 success of Strominger and Vafa in computing the entropy of particular $D = 5$ supersymmetric (BPS) black holes occurring in low-energy string theory, by using a conformal field theory appropriate to the microscopic physics of strings and D-branes, even raised the profile of string theory itself in a significant way. Further successes followed; the thermodynamic entropy of BPS and near-BPS black holes in various dimensions was reproduced successfully, in some cases even beyond leading order in macroscopic quantum numbers such as mass, charge, and angular momenta. A microscopic string theoretic accounting of the entropy of neutral black holes and black branes remains more elusive, however. One reason is that neutral black holes and branes are literally as far from BPS as possible, meaning that supersymmetry may provide no help at all in the endeavour.

BFSS matrix theory [4] is a conjectured relationship between quantum $D = 11$ M theory in light-front frame and the supersymmetric quantum mechanics of a large number N of D0-branes. This theory was recruited, e.g. in [5], to help explain the entropy of near-BPS black holes in dimensions $D = 10 - p$, and also in attempts to explain the entropy of the neutral cases as well. Reviews of this story include [6] (see references therein). Another interesting proposal was in [7]. Here, we simply record one major common feature of these various matrix theory approaches to counting the entropy of neutral black holes, which is relevant to our work here. Namely, the need for an infinite boost in the 11th dimension x^{11} in order to relate the neutral black holes of interest to nearly-BPS systems whose entropy can be computed microscopically using D-brane field theory. In particular, this infinite boost in the 11th dimension eliminates anti-D0-branes from the picture. The approach that we will take here will be different. Other interesting approaches to neutral black hole entropy from quite different perspectives include [8] and references therein.

Gravity/gauge correspondences are specific relationships between open string and closed string degrees of freedom. For the case of sixteen supercharges [9], they relate the degrees of freedom of supersymmetric Yang-Mills theories to closed string theory on near-horizon Dp-brane spacetimes. Subsequent string theoretic developments of interest for our work here include tachyon condensation in brane-antibrane systems. The paper which sparked our specific interest was [1], in which an interesting proposal was made for a microscopic explanation of the entropy of neutral black D3, M2 and M5 (non-dilatonic) branes in terms of a system of strongly coupled branes and antibranes.

In section 2, we review features of earlier analytic work, upon which we build here. Section 3 contains the bulk of our analytic observations. We study the entropy of neutral black Dp-brane spacetimes by using a microscopic model with an equal number of strongly coupled Dp-branes and anti-Dp-branes, with arbitrary angular momenta turned on. Our main result is that the supergravity entropy density can be reproduced by strongly coupled field theory, up to a power of two. Section 4 contains an exposition of preliminary numerical work on the $p = 0$ system. Our primary goal in the numerical part of our work is to try to test some assumptions of the brane-antibrane model, by doing direct strong-coupling simulation of the $p = 0$ system, adapting methods of

[2, 3]. We use a bosonic toy model to do this analysis, and find preliminary evidence that tachyon stabilization may indeed occur in the fashion expected from the work of [1]. We are not yet able, however, to be conclusive about the supersymmetric case.

While this paper was in preparation, the work of [21] appeared, which has some overlap with section 3, for the case $p = 3$ with angular momentum.

2 Analytic ingredients from prior work

2.1 Gravity/gauge correspondences for general p

For a system of N Dp-branes, it is possible[1] to take a clean low-energy limit such that the open strings with endpoints on the Dp-branes decouple from the closed strings in the bulk. This observation led, of course, to the celebrated AdS/CFT correspondence and the non-conformal open/closed-string correspondences of Itzhaki, Maldacena, Sonnenschein and Yankielowicz [9]. The theory on the branes is supersymmetric Yang-Mills theory (SYM) with sixteen supercharges. The 't Hooft coupling for the SYM theory is

$$g^2_{\mathrm{YM}}N \equiv (2\pi)^{p-2} g_s \ell_s^{p-3}\ N\,, \tag{1}$$

where g_s is the string coupling and $\ell_s \equiv \sqrt{\alpha'}$ is the string length.

These open/closed string correspondences tell us that we can hope to understand the gravitational fields of Dp-branes via the dual SYM theory. For $p \neq 3$ the fields describing the $D = 10$ geometry vary radially in the spacetime, and on the SYM side the coupling is dimensionful, yielding breakdown of the weak-coupling description either in the UV or the IR. In other words, there are limits to the validity of both descriptions. A dimensionless control variable is given by $g^2_{\mathrm{eff}} \equiv g^2_{\mathrm{YM}}NU^{p-3}$; on the supergravity side, $U \equiv r/\ell_s^2$ is the radial isotropic coordinate in energy units. The requirements [9] for the supergravity geometry in $D = 10$ to remain valid are

$$1 \ll (g^2_{\mathrm{YM}}NU^{p-3}) \ll N^{\frac{4}{(7-p)}}\,. \tag{2}$$

At the left-hand end, α' corrections become important and the SYM theory takes over from the $D = 10$ geometry as the weakly-coupled description; at the right-hand end, strong coupling (dilaton) ensues and it is necessary (for large-N) to turn to the S-dual supergravity geometry.

It was earlier noted in [10] that thermodynamic properties of near-BPS Dp-brane (and M-brane) geometries could be written in a way that is suggestive of a field theory interpretation. The main focus of that paper was the non-dilatonic branes, for which the energy density above extremality Δm and entropy density s were written in a way reminiscent of gases of weakly interacting massless particles in $d = p + 1$. For the dilatonic cases, however, the expressions do not yield recognizable weak-coupling results. The physical interpretation of this fact is that the strongly coupled physics of the $p \neq 3$ SYM theories gives rise to nontrivial dependence of the entropy and the energy above extremality on the temperature,

$$\Delta m(T) \propto T^{\frac{2(7-p)}{(5-p)}}\,, \qquad s(T) \propto T^{\frac{(9-p)}{(5-p)}}\,, \tag{3}$$

[1]For $p \leq 4$, which are the cases on which we will concentrate

where T is the Hawking temperature of the geometry.

In the $p = 0$ case, checking aspects of the open/closed-string correspondence conjectures is potentially feasible. The reason is of course that the SYM theory in this case is actually matrix *quantum mechanics* – albeit with sixteen supercharges. The numerical investigations of [2, 3] aimed to check the $p = 0$ correspondence explicitly, by finding the entropy of the system of *strongly coupled* N D0-branes and comparing it to the entropy of the near extremal D0-brane supergravity background. Results obtained showed that the 9/5 power in the expression for the entropy in (3) was indeed approximately reproduced in the numerical approach. This striking result, in combination with the work given a lightning review in the next subsection, provided the essential motivation for our work.

2.2 Brane and antibranes at finite temperature and the entropy of black branes

Motivated by an observation of Horowitz, Maldacena, and Strominger [11], Danielsson, Guijosa and Kruczenski [1] argued that, starting with a brane-antibrane system at zero temperature, turning on a finite temperature could lead to reappearance of open string modes. This physics was argued to ensue at strong open-string coupling $g_s N$ but weak closed-string coupling g_s, as would be appropriate to a model with validity in the black brane regime. Note that, to have this reappearance of the open string modes at temperatures *below* the Hagedorn temperature, going to the regime of strong open-string coupling was crucial. In alternative language, the tachyonic mode becomes stabilized by finite-temperature and strong coupling physics. Based on these observations, the authors of [1] formulated a microscopic brane-antibrane model for the entropy of non-dilatonic black branes: the D3, M2 and M5 cases.

The model has two SYM theories on worldvolumes, one for the branes and the other for the anti-branes. The tachyon, argued to be stabilized by finite-temperature and strong coupling effects, does not contribute to the total energy of the black hole, because it sits almost on top of the tachyon potential. It is also argued that the tachyon does not contribute to the total entropy, because the tachyon gets a large thermal mass.

The black holes of interest to us are neutral. Since the black 3-brane in $D = 10$ (or, equivalently, the black hole in $D = 7$) is neutral, it is taken to be modelled by the gas on a number N of D3-branes, the gas on an identical number N $\overline{\text{D3}}$-branes, and the tachyon dynamics. Since the branes and anti-branes appear in equal numbers, it makes sense to assume that the corresponding temperatures are equal. Using these assumptions, for the strongly coupled field theory side,

$$
\begin{aligned}
m_{\text{FT}} &= 2N\tau_3 + a\frac{\pi^2}{8}N^2T^4, \\
s_{\text{FT}} &= a\frac{\pi^2}{6}N^2T^3\,,
\end{aligned}
\tag{4}
$$

where τ_3 is the D3-brane tension and a is a constant known [12] from AdS/CFT to be 6 (not 8) when the SYM theory in question is strongly coupled. Note that the quantities of thermodynamic interest written here are specific, i.e. mass and entropy *densities*.

Rearranging to eliminate T gives

$$s_{\rm FT} = a(\pi^2/6)N^2 \left(\frac{m_{\rm FT} - 2N\tau_3}{a(\pi^2/8)N^2}\right)^{3/4} . \tag{5}$$

It is important to emphasize that this analysis is done in the *microcanonical* ensemble, where the total mass and charge of the black hole, as well as any angular momenta present, are kept fixed. Working in the microcanonical ensemble prevents thermal fluctuations from creating an infinite number of D3-$\overline{\rm D3}$ pairs. Working in canonical picture, by contrast, would allow the system to catastrophically create an infinite number of pairs, by extracting an arbitrary amount of energy from the reservoir, as it is a thermodynamically favourable state. In other words, one indefinitely spends energy to make more entropy, which makes the thermal ensemble destabilized.

The essential innovation in [1] is that the total number of branes is regarded as an independent variable, which is determined thermodynamically. The next step in the analysis, then, is to maximize the entropy with respect to N. The value of N so obtained is related to the mass density as

$$m_{\rm FT} = 5N\tau_3 \, . \tag{6}$$

To proceed further, it is necessary to decide what to do with the masses. One assumption is to take $m_{\rm FT} = m_{\rm SG}$. Using this, and substituting back for the optimized value of N leads to the final expression for the entropy from the strongly coupled field theory side,

$$s_{\rm FT} = a^{\frac{1}{4}} 2^{\frac{5}{4}} 3^{-\frac{1}{4}} 5^{-\frac{5}{4}} \pi^{\frac{1}{4}} \sqrt{\kappa} m_{\rm FT}^{\frac{5}{4}} \, , \tag{7}$$

where $\kappa = \sqrt{\pi}/\tau_3$. One the other hand, entropy of a black 3-brane is

$$s_{\rm SG} = 2^{\frac{9}{4}} 5^{-\frac{5}{4}} \pi^{\frac{1}{4}} \sqrt{\kappa} m_{\rm SG}^{\frac{5}{4}} \, . \tag{8}$$

Identifying $m_{\rm SG} = m_{\rm FT}$ and $a = 6$ gives

$$s_{\rm SG} = 2^{3/4} s_{\rm FT} \, . \tag{9}$$

This was the result of [1]. It shows that the entropy scaling from the brane-antibrane model is correct. It is worth noting here that this scaling agreement is nontrivial: it does *not* come about through simple dimensional analysis (and counting powers of N).

The overall coefficient misses, by a factor close to unity. It is perhaps not surprising, though, that the agreement, while close, is not exact. Reasons for this may include the fact that there is no clean decoupling between open string and closed string degrees of freedom. These effects not taken into account in the brane-antibrane model may indeed play a role, but they cannot be major effects for quantities like the entropy, because the scaling comes out correct.

3 Dp and $\overline{\rm D}p$-branes with angular momenta

In order to test the idea of the brane-antibrane model for neutral black hole entropy further, it is interesting to consider the cases other than $p = 3$, and to add angular momentum. This will be the main focus of the analytic part of our work.

It is convenient to begin by reviewing some salient properties of rotating black Dp-branes for various p. We are particularly interested in the $p=0$ story, since that is the case of the brane-antibrane model which we plan to test numerically, starting with the preliminary investigations of section 4.

3.1 Supergravity side

Black p-branes in $D=10$ are of course equivalent to $D\equiv 10-p$ dimensional black holes. We will not need the precise form of the supergravity fields; what is important for us here is the relationship between various physical parameters including tension, charge, angular momenta, and horizon radius. Using for example [13], and converting to quantities which are specific (per unit volume), we have for the $D=10$ black branes

$$\begin{aligned}
j_i &= \frac{2}{(7-p)}\frac{\alpha_p}{16\pi G}r_0^{7-p}\ell_i\frac{1}{\sqrt{1-\zeta^2}}\,,\\
T_H &= \frac{(7-p)-2\kappa}{4\pi r_H}\sqrt{1-\zeta^2}\quad\text{where}\quad \kappa=\sum_{i=1}^{n}\frac{\ell_i^2}{\ell_i^2+r_H^2}\,,\\
m &= \frac{\alpha_p}{16\pi G}r_0^{7-p}\left[\frac{1}{(7-p)}+\frac{1}{1-\zeta^2}\right]\,,\\
s &= \frac{4\pi}{(7-p)}\frac{\alpha_p}{16\pi G}r_0^{7-p}r_H\frac{1}{\sqrt{1-\zeta^2}}\,,\\
q &= \frac{\alpha_p}{16\pi G}r_0^{7-p}\left[\frac{\zeta}{1-\zeta^2}\right]\,,
\end{aligned}\tag{10}$$

where ζ is the boost rapidity parameter while r_0,ℓ_i are the original parameters used in creating the solutions, while the Newton constant is given by $16\pi G=(2\pi)^7g_s^2\ell_s^8$. We also use the shorthand $\alpha_p\equiv(7-p)\Omega_{8-p}$. Values of α_p are tabulated here for convenience

p	0	1	2	3	4	5	6
α_p	$7\pi^4/3$	$32\pi^3/5$	$5\pi^3$	$32\pi^2/3$	$6\pi^2$	8π	2π

The parameter r_H is the horizon radius given by

$$r_H^{7-p}\prod_{i=1}^{n}\left(1+\frac{\ell_i^2}{r_H^2}\right)-r_0^{7-p}=0\,.\tag{11}$$

It is straightforward to massage the expressions to write the entropy density s in terms of the energy density above extremality Δm. To do that, it is convenient use (11) to obtain r_0 in terms of r_H and ℓ_i. This is simple if we make the definitions

$$\rho\equiv\frac{r_H}{r_0}\quad,\quad\lambda_i\equiv\frac{\ell_i}{r_H}\,.\tag{12}$$

Then we have

$$\rho(\lambda_i)\equiv\frac{r_H}{r_0}=\left[\prod_{i=1}^{n}(1+\lambda_i^2)\right]^{-\frac{1}{(7-p)}}\,.\tag{13}$$

The energy density above extremality $\Delta m \equiv m - q$ is given by

$$\Delta m = \frac{\alpha_p}{16\pi G} r_0^{7-p} \left[\frac{1}{(1+\zeta)} + \frac{1}{(7-p)} \right] . \tag{14}$$

We can also write the Hawking temperature as

$$T_H = \sqrt{1-\zeta^2} \, \frac{(7-p)}{4\pi r_0} \frac{k_p(\lambda_i)}{\rho(\lambda_i)} , \tag{15}$$

where $\rho(\lambda_i)$ is given by (13) and we use the additional shorthand

$$k_p(\lambda_i) \equiv 1 - \frac{2}{(7-p)} \sum_{i=1}^{n} \frac{\lambda_i^2}{(1+\lambda_i^2)} . \tag{16}$$

In addition, the number of Dp-branes can be written as

$$N_p = \frac{\alpha_p}{16\pi G} \frac{1}{\tau_p} r_0^{7-p} \frac{\zeta}{1-\zeta^2} , \tag{17}$$

where

$$\tau_p = \frac{1}{g_s (2\pi)^p \ell_s^{p+1}} \tag{18}$$

is the Dp-brane tension.

Our main interest is to explain the entropy of neutral black branes in $D = 10$, or equivalently, neutral black holes in $D = 10 - p$. We therefore record the expressions here that we aim to reproduce using the strongly coupled brane-antibrane field theory model.

In the case of a neutral black hole, we have $\Delta m = m_{\rm SG}$. Defining

$$\delta_p \equiv \frac{(8-p)}{(7-p)} , \tag{19}$$

we have for the energy density

$$m_{\rm SG} = \frac{\alpha_p}{16\pi G} r_0^{7-p} \delta_p \tag{20}$$

and for the Hawking temperature

$$T_H = \frac{(7-p)}{4\pi r_0} \frac{k_p(\lambda_i)}{\rho(\lambda_i)} . \tag{21}$$

Therefore,

$$m_{\rm SG}(T_H) = \frac{\alpha_p}{16\pi G} \delta_p \left(\frac{(7-p)}{4\pi} \frac{k_p(\lambda_i)}{\rho(\lambda_i)} \frac{1}{T_H} \right)^{7-p} , \tag{22}$$

while for the entropy

$$s_{\rm SG}(T_H) = \frac{4\pi}{(7-p)} \frac{\alpha_p}{16\pi G} \rho(\lambda_i) \left(\frac{(7-p)}{4\pi} \frac{k_p(\lambda_i)}{\rho(\lambda_i)} \frac{1}{T_H} \right)^{8-p} . \tag{23}$$

Defining

$$a_p \equiv \frac{(7-p)}{2} \delta_p^{\delta_p} \left(\frac{\alpha_p}{16\pi} \right)^{\delta_p - 1} , \tag{24}$$

we have that

$$s_{\rm SG}(m_{\rm SG}) = \frac{2\pi}{Ga_p}\rho(\lambda_i)(Gm_{\rm SG})^{\delta_p}\,. \tag{25}$$

We may now ask how to unfurl the dependence of $\rho(\lambda_i)$ on $m_{\rm SG}$ and the rotation parameters j_i. The simplest way to proceed is to recognize that

$$\frac{\lambda_i}{2\pi} = \frac{j_i}{s}\,. \tag{26}$$

We therefore have the equation

$$\begin{aligned}\lambda_i\left[\prod_{i=1}^{n}(1+\lambda_i^2)\right]^{-\frac{1}{(7-p)}} &= a_p\frac{j_i}{G^{\delta_p-1}m_{\rm SG}^{\delta_p}} \\ &= \frac{j_i}{R_{\rm SG}(m_{\rm SG},G)}\,,\end{aligned} \tag{27}$$

where

$$R_{\rm SG} \equiv \frac{1}{a_p}G^{\delta_p-1}m_{\rm SG}^{\delta_p}\,. \tag{28}$$

Then, finally, we have

$$s_{\rm SG} = 2\pi\rho(R_{\rm SG}(m_{\rm SG},G),j_i)\,R_{\rm SG}(m_{\rm SG},G)\,. \tag{29}$$

We want to invert (27) to unfurl the dependence of λ_i on $m_{\rm SG}, G, j_i$. When no angular momenta are present, we have of course that $\rho = 1$. In general, however, the equation for λ_i is a polynomial of degree $(7-p)$ in λ, which is a quintic or worse for $p \leq 2$. For now, we do not concern ourselves with whether we can actually solve for ρ; we just leave the dependence of ρ on $(m_{\rm SG}, G, j_i)$ implicit. We also note that for $p=3$ and one angular momentum parameter j_1, we can actually solve to find

$$\lambda_1(p=3) = \sqrt{2\sqrt{\chi}\left(\sqrt{\chi}+\sqrt{1+\chi}\right)} \tag{30}$$

and

$$\rho(p=3) = \frac{1}{\sqrt{\sqrt{\chi}+\sqrt{1+\chi}}}\,, \tag{31}$$

where

$$\chi = 2^{-5}3^{-1}5^5\pi\frac{j_1^4}{G\,m_{\rm SG}^5}\,. \tag{32}$$

3.2 Near-extremal: the field theory side

In this subsection, we take the near-extremal limit of various supergravity formulæ to tell us how the strongly coupled field theory quantities should behave on the branes and anti-branes, using IMSY duality.

We should mention that in some respects our analysis here is quite similar to [10]. There are two major differences, however. The first is that we are interested in the effect of turning on angular momenta j_i. Our results must of course reduce to those of [10] upon taking $j_i = 0$. The second is that we plan to use the information about

the strongly coupled field theory, obtained via weakly coupled supergravity, using the techniques of [1].

Using the relation (17), we can[2] express r_0 in terms of N_p

$$r_0^{7-p} = 2(1-\zeta)\, N_p \tau_p \frac{16\pi G}{\alpha_p}\,, \tag{33}$$

(Of course, this r_0 is appropriate to the near-extremal geometry, and is not the same as the r_0 at the end of the previous subsection, which refers to the horizon radius of the *neutral* spacetime.) Equivalently,

$$2(1-\zeta) = \left(\frac{r_0}{\ell_s}\right)^{7-p} \frac{(2\pi\alpha_p)(2\pi)^{2(p-5)}}{\ell_s^{3-p}(g_{\rm YM}^2 N)}\,. \tag{34}$$

The energy density above extremality becomes

$$\Delta m_{\rm branes} = (1-\zeta) N_p \tau_p \frac{(9-p)}{(7-p)}\,. \tag{35}$$

Near extremality we have[3] for the Hawking temperature

$$\begin{aligned} T_H &= [2(1-\zeta)]^{\frac{(5-p)}{2(7-p)}} \left(\frac{\alpha_p}{16\pi G N_p \tau_p}\right)^{\frac{1}{(7-p)}} \frac{(7-p)}{4\pi}\frac{k_p(\lambda_i)}{\rho(\lambda_i)} \\ &= [2(1-\zeta)(2\pi)^4]^{\frac{(5-p)}{2(7-p)}} \left[\frac{(2\pi\alpha_p)}{(g_{\rm YM}^2 N)\ell_s^4}\right]^{\frac{1}{(7-p)}} \frac{(7-p)}{4\pi}\frac{k_p(\lambda_i)}{\rho(\lambda_i)}\,. \end{aligned} \tag{36}$$

We can obtain the equation of state for the near-extremal black Dp-brane by eliminating ζ in favour of T_H,

$$\begin{aligned} \Delta m_{\rm branes}(T_H) &= N^2 \left\{\gamma_p (2\pi)^2 \left(2\pi\alpha_p\right)^{-\frac{2}{(5-p)}}\right\} \times \\ &\times \quad \left(g_{\rm YM}^2 N\right)^{\frac{(p-3)}{(5-p)}} \left[\frac{4\pi}{(7-p)}\frac{\rho(\lambda_i)}{k_p(\lambda_i)} T_H\right]^{\frac{2(7-p)}{(5-p)}}\,. \end{aligned} \tag{37}$$

Defining the abbreviation

$$\gamma_p \equiv \frac{(9-p)}{2(7-p)}\,, \tag{38}$$

we have for the entropy density on the branes

$$\begin{aligned} s_{\rm branes}(T_H) &= N^2 \left\{\frac{4\pi}{(7-p)}(2\pi)^2 \left(2\pi\alpha_p\right)^{-\frac{2}{(5-p)}}\right\} \times \\ &\times \quad \left(g_{\rm YM}^2 N\right)^{\frac{(p-3)}{(5-p)}} \rho(\lambda_i) \left[\frac{4\pi}{(7-p)}\frac{\rho(\lambda_i)}{k_p(\lambda_i)} T_H\right]^{\frac{(9-p)}{(5-p)}}\,. \end{aligned} \tag{39}$$

This equation is of central importance; it encodes the equation of state for the system.

[2]The constant c_p familiar from the TASI-99 lectures of one of us is related to the quantities used here by $c_p g_s \ell_s^{7-p} = 16\pi G/\alpha_p$.

[3]For reasons discussed in [14] etc., the cases $p = 5, 6$ are more problematic to interpret. We therefore restrict ourselves from here on to the cases $p \leq 4$.

Now we come to the crucial step. Following the innovation in [1], we actually take the lead for the behaviour of the strongly coupled field theory on the branes and antibranes by using the near-extremal supergravity results. This is tantamount to using IMSY duality [9]. Here, for general p, our proposal to use IMSY duality for the $p \neq 3$ case in the model of type [1] is a more nontrivial step than in the $p = 3$ case where the field theory behaved in a simple way. This assumption involves some nontrivial physics; like [1], we are not taking into account the lack of a clean decoupling limit in the brane-antibrane model. Nonetheless, it is our point of view here that taking the near-extremal supergravity result seriously for the strongly coupled field theory on both the set of branes and the set of antibranes is exactly what we need.

We are particularly interested to investigate this story for the D0-$\overline{\text{D}}$0 case. The reason is that in the $d = 0 + 1$ case we have the hope of actually checking the above assumption explicitly, using a strong-coupling simulation. In particular, a significant motivation for our investigation of the non-conformal ($p \neq 3$) cases in the first place was the result of [2, 3] in which the equation of state was approximately reproduced numerically. We will begin developing the numerical story for the D0-$\overline{\text{D}}$0 case in the next section, but for now we work out the analytics.

For the open-string gas on (say) the set of Dp-branes, we have

$$s_{\text{branes}} = \frac{2\pi}{b_p}\rho(\lambda_i)\sqrt{N}g_{\text{YM}}^{\frac{(p-3)}{(7-p)}}\,(\Delta m)^{\gamma_p}\ , \tag{40}$$

where we have defined

$$b_p \equiv \frac{(7-p)}{2}\gamma_p^{\gamma_p}(2\pi)^{\frac{(p-4)}{(7-p)}}\alpha_p^{\frac{1}{(7-p)}}\ . \tag{41}$$

Now, for our model, recalling that we have the field theory on both the branes *and* antibranes, we have

$$m_{\text{FT}} = (2)N\tau_p + (2)\Delta m(T_{\text{FT}})\,. \tag{42}$$

BPS branes carry no macroscopic entropy, so we take the entropy of the strongly coupled field theory system representing the neutral black brane to be twice the entropy of the gas on each set of branes, $s_{\text{FT}}(T_{\text{FT}}) = (2)s(T_{\text{FT}})$. We find

$$s_{\text{FT}} = (2)\frac{2\pi}{b_p}\rho(\lambda_i)\sqrt{N}\,g_{\text{YM}}^{\frac{(p-3)}{(7-p)}}\left(\frac{m_{\text{FT}}}{(2)} - N\tau_p\right)^{\gamma_p}\ . \tag{43}$$

We now need to know how $\rho(\lambda_i)$ depends on variables of interest. We again use the trick of the previous subsection to write $\lambda_i = 2\pi j_i/s$. Then

$$\begin{aligned}\lambda_i\rho(\lambda_i) = \lambda_i\left[\prod_{i=1}^{n}(1+\lambda_i^2)\right]^{-\frac{1}{(7-p)}} &= \frac{b_p}{(2)}\frac{1}{\sqrt{N}}\,(g_{\text{YM}})^{\frac{(3-p)}{(7-p)}}\,j_i\left(\frac{m_{\text{FT}}}{(2)} - N\tau_p\right)^{-\gamma_p}\\ &= \frac{j_i}{R_{\text{FT}}(N, m_{\text{FT}}, g_{\text{YM}}, \tau_p)}\,,\end{aligned} \tag{44}$$

where

$$R_{\text{FT}}(N, m_{\text{FT}}, g_{\text{YM}}, \tau_p) = (2)\frac{1}{b_p}\sqrt{N}g_{\text{YM}}^{\frac{(p-3)}{(7-p)}}\left(\frac{m_{\text{FT}}}{(2)} - N\tau_p\right)^{\gamma_p}\ . \tag{45}$$

Referring back to the entropy equation (43), we have for general p and general angular momenta that

$$s_{\rm FT} = 2\pi\, R_{\rm FT}(N, m_{\rm FT}, g_{\rm YM}, \tau_p)\, \rho(R_{\rm FT}(N, m_{\rm FT}, g_{\rm YM}, \tau_p), j_i)\,. \tag{46}$$

At this stage, we may wonder whether the equations for the physical parameters λ_i in terms of $(j_i, m_{\rm FT}, N, g_{\rm YM}, \tau_p)$ can actually be solved explicitly analytically. For some p, they can. For other cases, including the $p = 0$ case of most interest to us, however, they cannot. For now, we will put this issue aside, and just proceed with the dependence of ρ on $R_{\rm FT}$ and thereby on $(N, m_{\rm FT}, g_{\rm YM}, \tau_p, j_i)$ implicit.

3.3 Comparing field theory and supergravity

We now take the field theory result of the last subsection and ask what happens when we optimize with respect to N, the number of branes (and anti-branes). For simplicity, we first ask how this story works without angular momenta. Later we add angular momenta back in.

Optimizing the entropy of the strongly coupled brane-antibrane theories w.r.t. N gives

$$N = \frac{m_{\rm FT}}{2\tau_p(1+2\gamma_p)} \quad \text{i.e.} \quad m_{\rm FT} = (2N\tau_p)2\delta_p\,. \tag{47}$$

The energy density in the brane gases is then

$$m_{\rm FT} - 2N\tau_p = (2N\tau_p)2\gamma_p\,. \tag{48}$$

Substituting these expressions for N and $m_{\rm FT}$ back into the expression for the entropy, and converting Dp-brane quantities into the Newton constant, we obtain

$$s_{\rm FT} = 2^{-\gamma_p}\frac{2\pi}{a_p}G^{\delta_p-1}m_{\rm FT}^{\delta_p}\,. \tag{49}$$

Two possible conclusions can be drawn from this. The first is that

$$m_{\rm FT} = m_{\rm SG} \quad \text{and} \quad s_{\rm FT} = 2^{-\frac{(9-p)}{2(7-p)}}\, s_{\rm SG} \tag{50}$$

Alternatively, we can conclude that

$$s_{\rm FT} = s_{\rm SG} \quad \text{and} \quad m_{\rm FT} = 2^{\frac{(9-p)}{2(8-p)}}\, m_{\rm SG}\,. \tag{51}$$

The second interpretation has an interesting conclusion. It says that the field theory energy, which is simply the energy on the Dp-branes plus the energy on the $\overline{\rm D}p$-branes, is not simply the supergravity energy, but there is a (suitably negative) binding energy

$$|m_{\rm binding}| = m_{\rm SG}\left(2^{\frac{(9-p)}{2(8-p)}} - 1\right)\,. \tag{52}$$

We find this interpretation the more attractive one. Binding energy can be expected in our model, because of the lack of a *clean* decoupling limit in our system between the open-string and closed-string modes.

Let us now add back the angular momenta and see if it affects our exposition of the basic physics of the brane-antibrane model.

Recall that, in the microcanonical ensemble that we are studying, both $m_{\rm FT}$ and j_i are constants. This is why we have labelled $R_{\rm FT}(N)$ a function of N, which we have to optimize following the model of [1].

Regardless of whether the equation for λ_i can be solved explicitly, the entropy depends only on $\rho(R_{\rm FT})$, as in (46). So we just proceed with $\rho(R_{\rm FT})$ implicit.

Our principle is to maximize the entropy as a function of N. We have

$$\begin{aligned} \frac{1}{s_{\rm FT}}\frac{\partial s_{\rm FT}}{\partial N} &= \frac{1}{R_{\rm FT}}\frac{\partial R_{\rm FT}}{\partial N} + \frac{1}{\rho(R_{\rm FT})}\frac{\partial\rho}{\partial R_{\rm FT}}\frac{\partial R_{\rm FT}}{\partial N} \\ &= \left[\frac{1}{R_{\rm FT}} + \frac{1}{\rho(R_{\rm FT})}\frac{\partial\rho}{\partial R_{\rm FT}}\right]\frac{\partial R_{\rm FT}}{\partial N}\,. \end{aligned} \tag{53}$$

Since we do not in general know the analytic dependence of ρ on $R_{\rm FT}$, we need to find the derivative implicitly. We have from (45)

$$\lambda_i \frac{1}{\rho}\frac{\partial\rho}{\partial R} = -\frac{j_i}{R^2\rho} - \frac{\partial\lambda_i}{\partial R}\,. \tag{54}$$

Using (46), and $\lambda_i/(2\pi) = j_i/s$, we have $\lambda_i = j_i/(\rho R)$, which gives

$$\lambda_i\left[\frac{1}{R_{\rm FT}} + \frac{1}{\rho(R_{\rm FT})}\frac{\partial\rho}{\partial R_{\rm FT}}\right] = -\frac{\partial\lambda_i}{\partial R_{\rm FT}} \tag{55}$$

Referring back to (44), it is easy to find $\partial\lambda_i/\partial R$,

$$\frac{\left[(7-p)+(5-p)\lambda_i^2\right]\lambda_i^{6-p}}{(1+\lambda_i^2)\prod_j(1+\lambda_j^2)}\frac{\partial\lambda_i}{\partial R_{\rm SG}} = -\frac{(7-p)}{R_{\rm SG}^{8-p}}j_i^{7-p} \tag{56}$$

Therefore the derivative is manifestly negative, as long as $p \le 4$, and it is nonzero as long as at least one angular momentum parameter (j_i) is turned on. Consequently, the term in square brackets in (53) does not vanish, and it is nonsingular. Therefore, whatever the behaviour of $\rho(R_{\rm FT})$, we are safe in concluding that the entropy is extremized by demanding that $\partial R_{\rm FT}(N)/\partial N = 0$. Optimizing $R_{\rm FT}(N)$ we find that

$$N = \frac{m_{\rm FT}}{2\tau_p(1+2\gamma_p)}\,. \tag{57}$$

which is exactly what we had for the non-rotating case.

Substituting back this optimal value of N into the field theory quantity $R_{\rm FT}$, we find that

$$R_{\rm FT}(m_{\rm FT}, j_{i\,\rm FT}, g_{\rm YM}, \tau_p) = R_{\rm SG}(m, j_{i\,\rm SG}, G)\,, \tag{58}$$

where $j_{i\,\rm FT}$ are the angular momenta in *one* copy of the strongly coupled field theory, if we make the identifications

$$m_{\rm FT} = m_{\rm SG} 2^{\frac{(9-p)}{2(8-p)}}\,, \quad j_{i\,\rm SG} = j_{i\,\rm FT}\,. \tag{59}$$

Now, naïvely we would have expected the total angular momenta of the neutral supergravity solution to be split in half, shared equally between the strongly coupled brane and antibrane field theories. The fact that the angular momenta and the mass do not match precisely is not particularly surprising, however, because of the lack of a clean decoupling limit. In fact, closed-string modes (whose physics not included in

the brane-antibrane model) might carry both mass and angular momenta. We find it intriguing, though, that the renormalization factors are simply powers of two!

Now, in order to get the entropy to match between the field theory side and the supergravity side here, we have the freedom to apply renormalization factors to both the mass and angular momenta. Alternatively, there is insufficient information to set these renormalizations factors unambiguously using our analysis.

Let us then accept the mass and angular momenta renormalizations of (59). We then find the remarkable fact that the functional form of the field theory entropy (46) in terms of R is identical to the functional form of the entropy on the supergravity side (29). With the renormalizations (59), we see that the R's are the same on both sides. The strongly coupled field theory of branes and antibranes therefore reproduces the supergravity result, up to renormalization factors of two, i.e.

$$s_{\rm FT}\left(m_{\rm FT} = 2^{\frac{(9-p)}{2(8-p)}} m_{\rm SG}, j_{i\,FT}\right) = s_{\rm SG}(m_{\rm SG}, j_{i\,SG})\,. \tag{60}$$

This is our main analytic result. It shows that, regardless of rotation or p, the entropy of the neutral black brane can be recovered from the strongly coupled brane-antibrane field theory. It is also worth noting that this result *cannot* be obtained just by dimensional analysis.

Alternatively, we can think of this result as constituting a highly nontrivial relationship between the thermodynamical properties of near-extremal black branes and those of neutral black branes.

Before moving on to develop some more physics of the brane-antibrane model, we may ask what our conclusion about the mass renormalization may do to other parts of our brane-antibrane-gas model. We know that

$$T_H^{-1} = \frac{\partial s_{\rm SG}}{\partial m_{\rm SG}}\,. \tag{61}$$

Therefore if, as we have shown, the entropies match but the masses are not equal, then the temperature is also affected in the same proportion as the mass

$$T_{\rm FT} = T_H 2^{\frac{(9-p)}{2(8-p)}}\,. \tag{62}$$

(Note that this does not spoil any of our previous assumptions.)

One might wonder how the unusual thermal properties of neutral black holes and black branes, such as negative heat capacity, could possibly be understood from analyzing this Dp-$\overline{\mathrm{D}p}$ field theory model, which is based on ordinary super-Yang-Mills systems in various dimensions. The explanation of this point in the context of the Dp-$\overline{\mathrm{D}p}$ model is simple and was given for the conformal cases in [1]. Upon inspection, one finds that there is a correlation between the energy in the open string gas and the contribution to the energy coming from the tension of the branes. Actually they are proportional to each other. This would give rise to the following interpretation for the negative specific heat. Namely, that since the energy in the gases is proportional to the tension energy, if we add more energy to the open string gases then we must create more Dp-$\overline{\mathrm{D}p}$ pairs to maintain the proportionality. This requirement to make more Dp-$\overline{\mathrm{D}p}$ pairs makes the system cool down, meaning that the more massive the black branes the colder they get. As we see, the moral of the story is that 'normal' SYM field theories on the worldvolume of the branes and antibranes behave 'abnormally', because

the total number of degrees of freedom controlled by N is not a constant; rather, it is given thermodynamically by the entropy maximization scheme.

So it is natural and important to ask whether the same kind of correlations also hold in our particular system of interest, i.e. D0-$\overline{\mathrm{D0}}$, and in an even broader sense, for a general Dp-$\overline{\mathrm{D}p}$ system.

It is easy to show the linear proportionality (48) between the energy in the gas and the energy contribution from the brane tension. Using data from the optimization of N

$$N \sim \frac{m_{\mathrm{FT}}}{\tau_p}\,, \tag{63}$$

where $m_{\mathrm{FT}} = 2N\tau_p + m_{\mathrm{gas}}$, gives

$$m_{\mathrm{gas}} \sim N\tau_p\,. \tag{64}$$

It is intriguing that exactly the same behaviour was observed in the case of non-dilatonic branes (D3, M2, M5). However, the way the result of [1] was obtained, comparing energy density and pressure in field theory and supergravity, does not apply to our $p = 0$ case of particular interest as there is no pressure on a $0 + 1$ dimensional worldvolume.

It is important to note that in our entire analysis, the black objects under study are neutral - nonperturbatively nonextremal. It is quite pleasing that the brane-antibrane model precisely explains the entropy of the neutral black branes and black holes, up to renormalizations of the mass and angular momenta that we computed.

3.4 Horizon size

It is interesting to ask whether the transverse fluctuations of the Dp-$\overline{\mathrm{D}p}$ system in the microscopic picture can reproduce the size of the horizon of the corresponding black brane geometry. We now do a scaling analysis, not keeping precise numerical factors.

Before addressing the general-p cases, it is instructive to review the simplest case $p = 3$. For N *near-extremal* D3-branes at strong coupling, power counting in N and conformal symmetry tell us that[4] $\langle \vec{X}^2 \rangle_{\mathrm{rms}} \sim NT^2$. Using the model of [1] to find the optimal value of N gives $N \sim m_{\mathrm{FT}}/\tau_3 \sim m_{\mathrm{gas}}/\tau_3$. Here we have used the information from the brane-antibrane model both to set the optimal value of N and to learn that there is roughly the same amount of energy density in the branes and in the open-string gas on those branes. Next, we use the assumption $m_{\mathrm{SG}} \sim m_{\mathrm{FT}}$, and the supergravity relationship between the Hawking temperature and the energy, $m_{\mathrm{SG}} \sim r_0^4/G$. We also recall the assumption that the brane gas temperature is equal to the antibrane gas temperature, and both are equal to the Hawking temperature of the black brane. Collecting these facts, and using the relationship $G\tau_3^2 \sim 1$, then gives a relationship between the supergravity horizon radius and variables in the field theory. The last assumption to be used is the supergravity relationship between the Hawking temperature and horizon radius $T_H \sim 1/r_0$. So $\langle \vec{X}^2 \rangle_{\mathrm{rms}} \sim r_0^4 (1/r_0)^2 \sim r_0^2$. Therefore, the r.m.s. extent of the position fields of the field theory corresponds to the horizon radius. In other words, we started from a near-extremal gauge theory and ended up with the size of the horizon of a *neutral* black 3-brane.

[4]The r.m.s. expectation value is normalized with a factor of $1/N$.

Now we turn away from the conformal case. In later sections, we will be particularly interested in the $p = 0$ case, so let us look at it here in some detail.

In the mean field approximation, it has been shown [15] that the extent of the ground state of the D0-brane theory at low temperatures is controlled by t'Hooft coupling, *not* the temperature. In supergravity, on the other hand, the rough estimate of the size *is* temperature dependent. Therefore, the above logic that we used for the D3-brane case will not work for the D0-brane case. In [15], a remedy for this confusing situation was proposed. Namely, that fast-varying degrees of freedom are physically inaccessible to a local supergravity observer. The supergravity probes just cannot resolve high-frequency fluctuations of order of t'Hooft energies, that are occurring at a "microscopic" level in quantum mechanics. In other words, a supergravity probe cannot resolve distances sharper than $l_{\rm probe} \sim \beta$. As a consequence, the supergravity probes simply miss all the quantum dynamics of the D0-brane dynamics involving energies much higher than the Hawking temperature. Therefore, in making a microscopic model to reproduce supergravity, caution must be used regarding which degrees of freedom have to be included in the picture.

Practically, the suggestion of [15] amounts to imposing a temperature-dependent cutoff on the spectral density of the transverse fluctuations, i.e. X^i propagators, that leads to a temperature dependent $\langle \vec{X}^2 \rangle_{\rm rms}$ which qualitatively shows the same behavior as r_0^2 does on supergravity side.

So in what follows we will assume that, if one tries hard from the gauge theory point of view, the extension of the wavefunction of the near-extremal geometry should become consistent with supergravity expectations. We use this intuition motivated by IMSY in what follows.

We now apply this thinking that came from studying D0-brane physics to the general-p case. We put together a number of ingredients that we have discussed in this section. We begin with

$$m_{\rm FT} \sim m_{\rm branes} + m_{\rm gas} \,, \tag{65}$$

and note from the model of [1] we have

$$m_{\rm gas} \sim m_{\rm branes} \,. \tag{66}$$

(Of course, the same energy density is in the brane gas and antibrane gas, so for the purposes of scaling we do not need to compute each contribution separately.)

From e.g. [9], we have for the spatial extent of a *near-extremal* Dp or $\overline{{\rm D}p}$ geometry,

$$U_0 \sim (g_{\rm YM}^2 N)^{1/(5-p)} T^{2/(5-p)} \,, \tag{67}$$

where $U_0 \equiv r_0/\ell_s^2$. (Thinking of U_0 as distance and T as energy, we see that this is exactly the same energy/distance relation found by [14] for a $D = 10$ supergraviton probe of a Dp-brane geometry in the decoupling limit. The coincidence is not too surprising.) We now add in dynamical information from the brane-antibrane model,

$$N \sim m_{\rm FT}/\tau_p \,, \tag{68}$$

to give

$$r_0^{5-p} \sim G m_{\rm FT} T^{2/(5-p)} \,. \tag{69}$$

Lastly, we bring in the (field theory) relationship between temperature and mass

$$T \sim (G m_{\rm FT})^{-1/(7-p)} \,. \tag{70}$$

Finally, we find

$$r_0 \sim (Gm_{\rm FT})^{1/(7-p)} \,. \tag{71}$$

Since $m_{\rm FT} \sim m_{\rm SG}$, this is the same radius as that of the horizon in the *neutral* geometry of interest. Therefore, the brane-antibrane model does successfully provide a consistent picture of the spatial extent of the horizon.

We now turn to checking the consistency of the supergravity approximation itself.

3.5 Validity of supergravity

The black Dp-branes on which we concentrate here are neutral. In order that they can be thought of as bona fide supergravity entities, we require at a minimum that the curvature at the horizon radius should be small in string units, to keep α' corrections small. In scaling,

$$(Gm)^{1/(7-p)} \gg \ell_s \,. \tag{72}$$

Using the equilibrium value of N, this becomes

$$(g_s^2 \ell_s^8 N \tau_p)^{1/(7-p)} \gg \ell_s \,, \tag{73}$$

which in turn leads to

$$g_s N \gg 1 \,. \tag{74}$$

Of course, we also require that string loop corrections be under control

$$g_s \ll 1 \,, \tag{75}$$

for the $D = 10$ neutral black brane spacetime.

Now, let us recall one important piece of physics from [1], the study of the conformal case. Even though the neutral geometry of interest is not studied in the decoupling limit, the brane and antibrane systems in the microscopic model are actually taken to be decoupled – as facts from AdS/CFT are used to describe the strongly coupled D3-brane theory. Therefore, at this point, it is appropriate to check whether the conditions for validity of the $D = 10$ near-extremal Dp-brane geometry are satisfied here also for the non-conformal cases, as these geometries dictate for us the behaviour of the strongly coupled field theories on the Dp and $\overline{\mathrm{D}p}$.

Let us therefore study the IMSY conditions carefully, to gain understanding of the brane-antibrane side of the picture. As described in [9], there is a region in which type II $D = 10$ supergravity is valid

$$1 \ll g_{\rm eff}^2(U) \ll N^{\frac{4}{(7-p)}} \,, \tag{76}$$

where the effective coupling at energy scale U is

$$g_{\rm eff}^2(U) \sim g_{\rm YM}^2 N U^{p-3} \,. \tag{77}$$

Also, the horizon radius for the near-extremal geometry is related to the energy above extremality $m_{\rm gas}$ and the temperature T by

$$U_0 \sim (g_{\rm YM}^4 m_{\rm gas})^{\frac{1}{(7-p)}} \sim (g_{\rm YM}^2 N)^{\frac{1}{(5-p)}} T^{\frac{2}{(5-p)}} \,. \tag{78}$$

Now, our big neutral black brane will have a low Hawking temperature. This means that we are not in danger of violating the α' (left-hand) end of the bound (76). However, precisely because we are operating at such low temperatures, we may be concerned about violating the strong-coupling (right-hand) end of the bound (76). For example, for $p = 0$, we may be concerned about having to lift up to $D = 11$.

It is a satisfying fact that our microscopic brane-antibrane model remains consistent in the $D = 10$ picture. The essential reason for this is that, in the brane-antibrane model, N is not an independent variable. Rather, N is thermodynamically determined. Using the fact that $m_{\rm gas} \sim N\tau_p$ and formulæ (77,78), and requiring that both ends of the IMSY bound (76) are respected gives two conditions, which reduce to

$$g_s N \gg 1\,, \qquad g_s \ll 1\,. \tag{79}$$

As we saw before, open strings are strongly coupled, but closed strings are weakly coupled.

Therefore, we do not need to be concerned about departing from the regime of validity of $D = 10$ supergravity, for our systems of strongly coupled branes and antibranes.

In particular, for the $p = 0$ case, we are always in the $D = 10$ supergravity regime. This means, in particular, that we never get to the $D = 11$ supergravity regime. Therefore, it is not apparent whether there is any simple relationship between these microscopic models for $D = 10$ neutral black branes and the BFSS matrix theory models, whose microphysical description is in terms of D0-brane degrees of freedom representing $D = 11$ M theory.

4 Numerically investigating D0$-\overline{\text{D0}}$ physics

For the remainder of this paper we will concentrate on the $p = 0$ case, i.e. the brane-antibrane model of the $D = 10$ Schwarzschild black hole. The reason is that the physics is a quantum mechanics, lending itself to the possibility of actually computing the behaviour of the strongly coupled field theory.

4.1 Scalings, and strategy

Of course, the idea of doing direct numerical simulations in the strongly coupled QM is not new. Kabat et al. [2] have used a method called Variational Perturbation Theory (VPT) to check the IMSY gravity/gauge duality conjectures [9] for the decoupling limit of Dp-brane systems with sixteen supercharges. In particular, the entropy of N near extremal D0-branes was computed using VPT, and was found to match the with supergravity result to the level of approximation used [3]. The agreement is impressive. In particular, the highly nontrivial temperature dependence of the free energy was obtained (approximately): $\beta F \propto T^{1.8}$! Indeed, this could be considered as the first (approximate) nonperturbative check of any gauge theory/gravity duality. The causal structure of spacetime from the point of view of the gauge theory also has been studied in the context of mean field Gaussian approximations [16].

The next step for our program to understand $D = 10$ Schwarzschild black holes would be to justify the crude brane-antibrane picture drawn in the last few sections, by

computing the microscopic entropy using the effective quantum description of a system of D0-$\overline{\rm D0}$s.

Now, in the D0-$\overline{\rm D0}$ system we cannot take a clean decoupling limit, if we expect to keep the open string tachyon and massless string modes but not massive string modes. Therefore, our further progress in developing the brane-antibrane model is to be thought of as an approximate description, where the massive string modes are not fully decoupled. Of course, the same story was true of the Danielsson et al. work [1].

Clearly, there are two different energy scales set by two different dimensionful coupling constants in our problem: α' and $g_{\rm YM}^2 = g_s(2\pi)^{-2}\ell_s^{-3}N$. In gauge theory, everything is governed by $g_{\rm YM}^2$, while α' controls the mass of the tachyon. For the physics of D0-branes alone, dimensional analysis and power counting in N tells us [2] that a dimensionless quantity like βF can be written as

$$\beta F_{\rm D0} \propto N^2 \mathcal{F}(\frac{T}{(g_{\rm YM}^2 N)^{1/3}})\,. \tag{80}$$

Here, with the tachyon, we have two different scales: the t'Hooft coupling and α'. Thus, the free energy can be written as

$$\beta F_{{\rm D0}-\overline{\rm D0}} \propto N^2 \mathcal{G}(\frac{T}{(g_{\rm YM}^2 N)^{1/3}}, \alpha'(g_{\rm YM}^2 N)^{2/3}) = N^2 \mathcal{G}(\frac{T}{(g_{\rm YM}^2 N)^{1/3}}, (g_s N)^{2/3})\,. \tag{81}$$

Therefore, the free energy written in dimensionless units will be a function of $g_s N$. Of course, it will also depend on the dimensionless inverse temperature measured in 't Hooft units, $\tilde{\beta}$.

In the limit where massive open string excitations can be neglected, and the string coupling is small, the system is relatively simple. We have two copies of the $D0$-brane theory - one for D0-branes and one $\overline{\rm D0}$-branes - plus a complex tachyon T and a massless Majorana fermion Ψ coming from D-$\overline{\rm D}$ open strings.

The strategy is to compute the free energy of this system as a function of the tachyon classical background field, plus other quantities at finite inverse temperature β. In the field theory we compute at strong couplings using VPT, and in the closed string description the $D = 10$ supergravity approximation is valid. Then the following pieces of information could be read off immediately from the free energy

Tachyon static mass The thermodynamically favourable value for the tachyon expectation value can be computed by minimizing the effective action, i.e. the free energy of the system as a function of the tachyon background.

Sign of the Tachyon dynamical mass Another important piece of information encoded in the free energy is the effective mass of the tachyon which is given by

$$m_T^2 = \beta \frac{\delta^2 F}{\delta\langle T\rangle \delta\langle T\rangle^\star}\,, \tag{82}$$

where $\langle T\rangle$ is the tachyon expectation value. This effective mass includes the contributions from infinite numbers of loop diagrams via the Schwinger-Dyson equation.

Magnitude of the Tachyon Dynamical Mass This quantity is a measure of the smallness of the tachyon fluctuations. A large dynamical mass results in small contributions to the entropy from the tachyon.

Phase Portrait of the theory The final goal is to calculate the phase portrait of the system, as a function of inverse temperature β and $g_s N$. A sign change in m_T^2 from negative to positive would be of great interest, since it would signal the tachyon stabilization phenomenon. This stabilization would give a justification for why a gas of D0 and $\overline{\mathrm{D0}}$ branes does not simply annihilate all the way down to a bunch of closed strings.

One note of caution. The Hagedorn phenomenon in string theory might impact us here in one place: at temperatures of order of the string scale, massive open string excitations become important, but we are not incorporating massive excitations into our dynamics. In order to have a self-consistent description of the phenomena, therefore, we will look for a possible finite-temperature tachyon stabilization at a temperature well below the Hagedorn temperature.

We now turn to describing the technology which we will use to perform the numerical simulation of the D0-$\overline{\mathrm{D0}}$ system.

4.2 Variational perturbation theory (VPT)

The basic idea of VPT is simple. Using VPT to simulate strong-coupling D0-brane physics was first introduced in [2, 3], and we review the salient points here for refernence. Suppose that the theory of interest has action S, and the aim is to approximate its free energy at *strong couplings*. The idea is to use a free theory, with action S_0, with arbitrary tunable parameters. The next step is to find values for these parameters in such a way the free theory is a best fit for the full interacting theory.

For any arbitrary S_0 the following identity holds

$$\beta F = \beta F_0 - \langle e^{-(S-S_0)} - 1\rangle_{0,C}\,, \tag{83}$$

where 0 refers to the free theory and C stands for connected contributions. Expanding the identity,

$$\beta F = \beta F_0 + \langle S - S_0\rangle_0 - \frac{1}{2}\langle (S-S_0)^2\rangle_{0,C} + \cdots\,. \tag{84}$$

It is important to note that this expansion is *not* a perturbative expansion in the couplings of the original interacting theory of interest.

Now, if terms to all orders in the above expansion are kept, then there is of course no dependence on the parameters of the trial free theory, as it is an identity true for any S_0. Taking a practical approach and terminating the series at any finite order, however, the series depends on the variational parameters.

The next step is to fix the variational parameters. Minimizing the free energy with respect to those parameters (which is equivalent to requiring the trial free action to satisfy the Schwinger-Dyson equation) leads to a set of algebraic coupled equations (in general infinite in number), called "gap equations". Solution of the gap equations yields the variational parameters, which are then substituted back to obtain the free energy.

This VPT method has in fact been checked explicitly for quantum mechanical systems where calculations in the full interacting theory can actually be done exactly, and the above expansion captures the strong-coupling behaviour accurately; in particular, convergence is very fast. Our system of interest here is of course not solvable, so we need to use the approximate expansion procedure outlined above.

Practically speaking, of course, the infinite set of equations cannot be solved. Therefore it is necessary to cut off that infinite set of equations, and find the common roots of the finite coupled set of algebraic equations. Solving this system of equations is, at any rate, computationally very expensive.

The last step is then to substitute the solution of the gap equations back into the free energy, to get the sought-after dependence on the parameters of interest. For us, these parameters will be the dimensionless inverse temperature $\tilde{\beta}$, and the open-string coupling $g_s N$.

As outlined in [2, 3], the VPT method is not straightforward when dealing with supersymmetry and gauge theory. Regarding gauge theory. Proposing a Gaussian theory for a non-dynamical gauge field in 0+1 dimensions is delicate, as one cannot simply gauge it away at finite temperature. Gauging away the gauge field leaves an observable (a Wilson loop), made from the zero mode of the gauge field encircling the Euclidean time direction, as a remnant. The Gaussian theory to be used [2] is the one-plaquette model studied by Gross and Witten [17]. Taylor-Slavnov identities, which are consequences of gauge symmetry and relate various correlation functions, get invalidated by the VPT expansion. Regarding supersymmetry. It turns out to be impossible in general to come up with a Gaussian theory with general variational parameters respecting supersymmetry, without inclusion of a trial action for auxiliary fields. A way to get around the problem is to work with the off-shell superspace formulation.

On the other hand, the VPT method has some special positive features as well. One is that VPT automatically respects 't Hooft counting. Another is that VPT automatically cures infrared problems arising from having an infinite moduli space of vacua where D0-branes are far apart. The contribution coming from the zero mode sector goes like $\mathcal{O}(N)$ and is therefore distinguishable from $\mathcal{O}(N^2)$ contributions.

4.3 Motivating the action

According to [18] (see also [19] for a similar results), the action for a system of D9-$\overline{\text{D9}}$ with the $U(1)$ gauge fields living on its world-volume is given by the following[5]

$$\begin{aligned} S &= -2\tau_9 \int d^{10}x \, \exp(-2\pi\alpha' T^\dagger T)\Big[1 + 8\pi\alpha'^2 \ln(2) D_\mu T^\dagger D^\mu T + \\ &\qquad \frac{(2\pi\alpha')^2}{8}\overline{F}_{\mu\nu}\overline{F}^{\mu\nu} + \frac{(2\pi\alpha')^2}{8} F_{\mu\nu}F^{\mu\nu} \ldots\Big], \end{aligned} \tag{85}$$

where we have used 'bars' to denote quantities in the $\overline{\text{D0}}$ sector and 'no bars' to denote quantities in the D0 sector. Of course, T is a complex bi-fundamental gauge field so that $D_\mu T = \partial_\mu T - iTA_\mu + i\overline{A}_\mu T$. Dimensional reduction of this action should give the correct action for the dynamics of lower dimensional Dp-$\overline{\text{D}p}$ systems including our $p = 0$ case. As usual, we substitute $\overline{A}_i = (2\pi\alpha')^{-1}\overline{X}_i$, $A_i = (2\pi\alpha')^{-1}X_i$. Performing

[5]We are using the mostly plus convention for the metric signature.

field redefinitions, a nonabelian generalization of this toy approximate action is

$$
\begin{aligned}
S &= -\int dx^0 \mathrm{Tr}\, \exp\left(-\frac{\pi^2\alpha'}{4\ln 2}T^\dagger T\right)\left[\frac{1}{2g_{\rm YM}^2}(\partial_0 T^\dagger\partial^0 T + i\partial_0 T^\dagger(\overline{A}^0 T - TA^0)\right.\\
&\quad + i(A^0T^\dagger - T^\dagger\overline{A}^0)\partial_0 T - (A^0T^\dagger - T^\dagger\overline{A}^0)(\overline{A}_0 T - TA_0)\\
&\quad \left. -(X^iT^\dagger - T^\dagger\overline{X}^i)(\overline{X}_i T - TX_i)) + 2\tau_0\mathbb{I}_{N\times N}\right]\\
&\quad + S'_{\rm D0}[X^i, A^0] + S'_{\overline{\rm D0}}[\overline{X}^i, \overline{A}^0]\,,
\end{aligned} \tag{86}
$$

where we use the usual definition of the 't Hooft coupling, $1/g_{\rm YM}^2 = 4\pi^2 g_s^{-1}\ell_s^3$, and where $S'_{\rm D0}$ and $S'_{\overline{\rm D0}}$ are the corresponding low-energy actions for D0 and $\overline{\rm D0}$ respectively. Obviously, T is in the $N\times\overline{N}$ bifundamental representation.

Now, in order to be able to compute the thermal partition function, we need to Euclideanize the Minkowskian action. First, we analytically continue the timelike component of both $U(N)$ gauge fields as well as Minkowskian time direction x^0 while leaving the other fields untouched,

$$
x^0 = -i\tau\,, \quad iS_E = S_M\,, \quad iA^0_E = A^0_M\,, \quad i\overline{A}^0_E = \overline{A}^0_M\,. \tag{87}
$$

Next, we Fourier expand T, A^0, $\overline{A}^0$, X^i and $\overline{X}^i$ in τ with a periodicity β given by the the inverse temperature. Since all these variables are bosonic, they have periodic boundary conditions,

$$
\left\{A_0(\tau), \overline{A}_0(\tau), X^i(\tau), \overline{X}^i(\tau), T(\tau)\right\} = \frac{1}{\sqrt{\beta}}\sum_{\ell\in\mathbb{Z}}\left\{A_{0\ell}, \overline{A}_{0\ell}, X^i_\ell, \overline{X}^i_\ell, T_\ell\right\} e^{\frac{-2\pi i\ell}{\beta}\tau}\,. \tag{88}
$$

Writing the action in terms of Fourier modes is straightforward. We take the above expansions and perform the Euclidean time integral, using the following *variational* correlators in Wick contractions of Feynman diagrams contributing to the free energy (in our case, the amplitude $\langle S_{IIE} - S_{0E}\rangle_0$,)

$$
\begin{aligned}
\langle X^i_{\ell AB} X^j_{mCD}\rangle_0 &= \sigma^2_{\alpha\ell}\delta^{ij}\delta_{\ell+m}\delta_{AD}\delta_{BC} \qquad i,j = 1,2\,,\\
\langle X^a_{\ell AB} X^b_{mCD}\rangle_0 &= \Delta^2_{\alpha l}\delta^{ab}\delta_{\ell+m}\delta_{AD}\delta_{BC} \qquad a,b = 3..9\,,\\
\langle A_{00AB} A_{00CD}\rangle_0 &= \rho_0^2\delta_{AD}\delta_{BC}\,,\\
\langle T^\dagger_{\ell AB} T_{mCD}\rangle_0 &= \xi^2_\ell\delta_{\ell m}\delta_{AD}\delta_{BC}\,.
\end{aligned} \tag{89}
$$

and similarly for the barred variables. The need for separation of transverse scalar directions comes from the limitations of the original superspace formulation [3] of the D0-brane supersymmetric matrix quantum mechanics. Namely, for each copy of the QM, there are 2 scalars X^i in the gauge multiplet and 7 other ϕ^a scalars coming from scalar multiplet. In other words, the original $SO(9)$ R-symmetry is broken to $SO(2)\times\ G_2$ (ϕ^a are a **7** of G_2).

The numerical simulation of this action, which is an approximation to the system of D0-branes, anti-D0-branes and the tachyon sector where both copies of the D0-brane theory are supersymmetric is computationally very expensive, partly because of the need to find the solution to hundreds of nonlinear equations. In addition, we have to cover mapping out of the phase portrait as a function of β and g_sN. We therefore report on some initial results in a more stripped-down version of our model.

4.4 The toy model

In this section, we look at a "toy" model which consists of large-N bosonic matrix quantum mechanics (representing the D0-brane and anti-D0-brane theories) plus our bi-fundamental charged tachyon.

This toy model is obviously a good approximation to the original problem in the high-temperature limit where the Euclidean time circle is so small that nonzero thermal KK modes are heavy. In this limit, there is no antiperiodic fermion left after reduction to $0+0$ dimensions. Of course, we are interested in low temperatures where the supergravity description is valid and quantum mechanics is strongly coupled.

There are reasons to believe why this toy model could demonstrate some generic behaviors of interest to us, which will continue to be true even in the case where we have two copies of *supersymmetric* matrix quantum mechanics. It is well known [20] that the extent of the ground state of a system of D0-branes, defined in terms of fluctuations of transverse scalars, is controlled by the 't Hooft coupling. It is intuitively plausible to expect that, at strong coupling, the tachyon could acquire a large induced mass through its coupling to these large transverse scalar fluctuations. As has been observed in [2], both bosonic matrix quantum mechanics and its supersymmetric version have large transverse fluctuations. So if one takes this intuition seriously, as far as the dynamics of the tachyon is concerned in the low energy approximation, even the bosonic version could be a good approximation to the right physics[6]

In the model of Danielsson et al [1] which motivated our work here, the dynamical assumption is that the tachyon field is stabilized by finite-temperature strong-coupling effects about $T=0$. We therefore expand $T=0+t$, where t is the fluctuation part, and plan to show the consistency of this assumption *a posteriori* by showing that the mass is indeed positive – and large[7] – at strong coupling. Therefore, the terms relevant to our discussion which are quadratic in the tachyon fluctuations t can be written

$$\begin{aligned} S_E &= \frac{1}{g_{\rm YM}^2}\int_0^\beta d\tau(\frac{1}{2}{\rm Tr}D_\tau X^i D_\tau X^i - \frac{1}{4}{\rm Tr}[X^i,X^j][X^i,X^j]) \\ &+\frac{1}{g_{\rm YM}^2}\int_0^\beta d\tau(\frac{1}{2}{\rm Tr}D_\tau \overline{X}^i D_\tau \overline{X}^i - \frac{1}{4}{\rm Tr}[\overline{X}^i,\overline{X}^j][\overline{X}^i,\overline{X}^j]) \\ &+\int_0^\beta d\tau{\rm Tr}[\frac{1}{2g_{\rm YM}^2}\Big(\partial_\tau t^\dagger\partial_\tau t + A^0 t^\dagger t A^0 + t^\dagger\overline{A}^0\overline{A}^0 t + X^i t^\dagger t X^i + t^\dagger \overline{X}^i\overline{X}^i t \\ &-\partial_0 t^\dagger(\overline{A}^0 t - tA^0) + (A^0 t^\dagger - t^\dagger\overline{A}^0)\partial_0 t\Big) + 2\tau_0\mathbb{I}_{N\times N} - \frac{2\tau_0\pi^2\alpha'}{4\ln 2}t^\dagger t\Big]. \end{aligned} \quad (90)$$

Morally, we are going to treat the tachyon as background. By this we simply mean that we are interested in finding the free energy as a function[al] of the tachyon, after integrating out the other fields in the problem – nonperturbatively using VPT. For this reason, no trial action for the tachyon is introduced; the VPT techniques are used for the fields other than the tachyon.

[6] Although irrelevant to the above discussion, the difference is in the actual value of the ground state energy, which is – for each copy – of course zero in the supersymmetric case but large and positive in the bosonic case.

[7] We have ignored the higher powers in the expansion of the exponential prefactor in (86) sitting in front of the Lagrangian; this will be justified a posteriori by the large-mass finding.

Using VPT, we have

$$\beta F = \beta F_0 + \beta \overline{F}_0 + \langle S_{\mathrm{D0}} + S_{\overline{\mathrm{D0}}} + S_T - S_0 \rangle_0 - \frac{1}{2}\langle (S_{III})^2 \rangle_{0,C} \quad (+\ldots)\,, \tag{91}$$

where S_T is the tachyon contribution to the action, and the subscript zero refers to the contribution coming from the trial free action for the other fields.

As a first step, we can neglect the terms in the action that are cubic in the tachyon, as far as the VPT expansion is concerned. There is a good reason why this is justified. Namely, because the cubic contribution to the free energy enters at the order of $\mathcal{O}(t^4)$, which is small at small-t.

Following the story for a single clump of D0-branes [2], we put in the ghost field from gauge-fixing, whose Fourier modes are denoted $s_{\ell\neq 0}$ (obviously, for physical reasons there is no zero mode). We also use equations from pages 25-27 of that work; to save space here we only write here what changes when we introduce our tachyon field.

We also introduce convenient dimensionless units. These are defined such that in term of them one gets an overall factor of N^2 in the free energy, and the rest is just a function of dimensionless quantities and $g_s N$. We define newly dimensionless variables with tildes; we have

$$\tilde{\beta} = \beta f^{1/3}\,, \quad \tilde{\rho}_0^2 = \frac{N\rho_0^2}{f^{1/3}}\,, \quad \tilde{\sigma}_l^2 = \frac{N\sigma_l^2}{f^{1/3}}\,, \quad \tilde{t}_\ell^\dagger \tilde{t}_\ell = \frac{N t_\ell^\dagger t_\ell}{f^{1/3}}\,, \tag{92}$$

and similarly for the barred variables. Here $f = (g_{\mathrm{YM}}^2 N)^{1/3}$. Note that in this bosonic model we do not have to treat the $X^i, \overline{X}^i$ fields for $i = 1, 2$ differently than for $i = 3\ldots 9$ because there is no need to worry about maintaining explicit supersymmetry; therefore we do not have Δ_ℓ and we just use σ_ℓ to represent correlators of all nine position fields. Note that we are measuring the tachyon expectation value in string units.

Henceforth, we drop the tildes for notational simplicity. Then we obtain the following expression for the free energy as a function of the tachyon

$$\begin{aligned}
\beta F \;=\;& \beta F(\lambda)_\square + \beta F(\overline{\lambda})_\square + \frac{N}{\lambda}\langle \mathrm{Tr}(U + U^\dagger)\rangle_\square + \frac{N}{\overline{\lambda}}\langle \mathrm{Tr}(\overline{U} + \overline{U}^\dagger)\rangle_\square \\
& - \frac{9N^2}{2}\sum_\ell \log \sigma_\ell^2 - \frac{9N^2}{2}\sum_\ell \log \overline{\sigma}_\ell^2 + N^2 \sum_{\ell\neq 0} \log s_\ell + N^2 \sum_{\ell\neq 0}\log \overline{s}_\ell \\
& + \frac{9N^2}{2}\sum_\ell ((\frac{2\pi l}{\beta})^2 \sigma_\ell^2 - 1) + \frac{9N^2}{2}\sum_\ell ((\frac{2\pi l}{\beta})^2 \overline{\sigma}_\ell^2 - 1) \\
& + N^2 \sum_{\ell\neq 0} ((\frac{2\pi l}{\beta})^2 s_\ell + 1) + N^2 \sum_{\ell\neq 0}((\frac{2\pi l}{\beta})^2 \overline{s}_\ell + 1) + \frac{9N^2}{\beta}\rho_0^2 \sum_\ell \sigma_\ell^2 \\
& + \frac{9N^2}{\beta}\overline{\rho}_0^2 \sum_\ell \overline{\sigma}_\ell^2 + \frac{36N^2}{\beta}(\sum_\ell \sigma_\ell^2)^2 + \frac{36N^2}{\beta}(\sum_\ell \overline{\sigma}_\ell^2)^2 \\
& + N^2 \sum_\ell \frac{1}{2}(\frac{2\pi l}{\beta})^2 \mathrm{Tr}\frac{t_\ell^\dagger t_\ell}{N^2} + \frac{N^2}{2\beta}(\rho_0^2 + \overline{\rho}_0^2)\sum_\ell \mathrm{Tr}\frac{t_\ell^\dagger t_\ell}{N^2} \\
& + \frac{9N^2}{2\beta}(\sum_\ell \sigma_\ell^2 + \sum_\ell \overline{\sigma}_\ell^2)\sum_\ell \mathrm{Tr}\frac{t_\ell^\dagger t_\ell}{N^2} \\
& - \frac{(2\pi)^{4/3} N^2}{8\ln 2 (g_s N)^{2/3}} \sum_\ell \mathrm{Tr}\frac{t_\ell^\dagger t_\ell}{N^2} + \frac{2(2\pi)^{2/3} N^2 \beta}{(g_s N)^{4/3}}\,.
\end{aligned} \tag{93}$$

In what follows, we use a natural ansatz for the propagators

$$\sigma_\ell^2 = \left((\frac{2\pi\ell}{\beta})^2 + m^2 \right)^{-1} , \tag{94}$$

and similarly for the barred variables.

The gap equations are easily obtained. We have (and similarly for barred variables)

$$m^2 = \frac{2\rho_0^2}{\beta} + \frac{16}{\beta}\sum_\ell \sigma_\ell^2 + \frac{1}{\beta}\sum_\ell \frac{t_\ell^\dagger t_\ell}{N^2} , \tag{95}$$

where ρ_0 can be explicitly obtained in terms of λ, the coupling in the one-plaquette action, because the one-plaquette theory is soluble in the large-N limit.

For $\lambda \leq 2$,

$$-1 + \frac{2}{\beta}\left(\frac{18}{\beta}\sum_\ell \sigma_\ell^2 + \frac{1}{\beta}\sum_\ell \mathrm{Tr}\frac{t_\ell^\dagger t_\ell}{N^2}\right)\left[1 - (1-\frac{2}{\lambda})\log(1-\frac{\lambda}{2})\right] = 0 , \tag{96}$$

while for $\lambda \geq 2$,

$$\lambda = \beta / \left(\frac{18}{\beta}\sum_\ell \sigma_\ell^2 + \frac{1}{\beta}\sum_\ell \mathrm{Tr}\frac{t_\ell^\dagger t_\ell}{N^2}\right) . \tag{97}$$

We will solve this system of highly nonlinear, coupled equations. The strategy will be to treat the tachyon as a background. Solving the gap equations, and substituting back the solution as a function of the tachyon, then gives the free energy as a function of the (background) tachyon. Given this effective action for the tachyon, it is then straightforward to read off the tachyon effective mass. The plan will be to work out the two-dimensional "phase portrait" of the system (with coordinates $g_s N$ and β), and find out whether any tachyon stabilization is possible. By this we simply mean that there are points in this two-dimensional portrait where the effective tachyon mass is positive.

Before we move to displaying our numerical results, a number of remarks are in order.

- As we saw earlier, in the supergravity limit in which we are interested, $g_s N \gg 1$. By inspection of the equations it might then seem that, in the units we use, the wrong-sign mass term for the tachyon is suppressed and we are done! This quick-and-dirty reasoning is, however, too naïve. In fact, in order not to excite any massive open string state, the temperature must be small in string units. In turn, this implies that

$$\beta \gg (g_s N)^{1/3} \gg 1 , \tag{98}$$

  where β is the dimensionless inverse temperature. With such low dimensionless temperatures, it becomes more and more difficult to overcome the wrong-sign mass term through the tachyon couplings to the transverse coordinates $X^i, \overline{X}^i$ and the world-volume gauge fields $A_0, \overline{A}_0$.

- It is crucial to remember that all variational parameters are *implicit* functions of the tachyon t. So it is certainly not true that we can simply read off the effective tachyon mass from the the coefficients of the quadratic tachyon fluctuation in the

expression for the free energy. For example, terms like $(N^2/\beta)\rho_0^2 \sum_\ell \sigma_\ell^2$, which do not have any *explicit* background tachyon dependence, are becoming tachyon-dependent through the gap equations. All such terms make contributions to the effective tachyon mass, as one expands the variational parameters around $\sum_\ell \mathrm{Tr} t_\ell^\dagger t_\ell = 0$. (Note that these expansions exist, since the solution to the gap equations is differentiable with respect to t in the vicinity of $\sum_\ell \mathrm{Tr} t_\ell^\dagger t_\ell = 0$.)

- Obviously, it is always possible to stabilize the tachyon at weak couplings, but the temperatures needed for this are above the Hagedorn temperature. It is easy to see this in perturbation theory, partly because at such high temperatures the D0 QM is perturbative. Perturbatively, to stabilize the tachyon, it is sufficient to have
$$T > \frac{1}{(g_s N)^{4/3} \ell_s} \gg \frac{1}{\ell_s}\,, \tag{99}$$
for small $g_s N$.

In our investigations, we use the Metropolis algorithm. This is a fast method generally; it really comes into its own for numerical investigations of the supersymmetric model, the numerical results from which we hope to report on in the near future. We summarize some important features of the Metropolis algorithm in the Appendix.

4.5 Numerical results

We are interested in the sign of the mass of the tachyon zero mode, for each point in the two-dimensional parameter space, coordinatized by $g_s N$ and β which is the dimensionless inverse temperature in 't Hooft units.

We now plot our results for the phase portrait of the toy model theory. Inverse temperature, in dimensionless 't Hooft units, is on the y-axis, while the open-string coupling $g_s N$ on the x-axis. (As a reminder, the parameter $g_s N$ tells us the bare negative mass-squared of the tachyon.) Points at which the tachyon mass squared is positive are indicated by a cross, and those at which it is negative are indicated by a circle. The green curve delineates where the Hagedorn phenomenon is important; points above the curve are below the Hagedorn temperature. The red straight line delineates the region where the gauged QM theory has a strong effective coupling (set by the inverse temperature and 't Hooft coupling); the temperatures where the quantum mechanical theory is strongly coupled lie above the red line.

For clarity, we separate the regimes of smaller $g_s N$ from the regime of larger $g_s N$.

As we expected for tachyon stabilization, at small $g_s N$, one has to go to very high temperatures; this is clear from Figure 1. As we see for large $g_s N$, on the other hand, even for low temperatures compared to the Hagedorn temperature, there is a possibility of tachyon stabilization.

5 Discussion

In this paper, following the lines of [1], we have examined the validity of the brane-antibrane model for neutral black branes of various dimensionality. What we have found is that this Dp-$\overline{\mathrm{D}p}$ picture works for arbitrary p. Interestingly, this is true even for the "dilatonic" branes, where the supergravity entropy cannot be written in

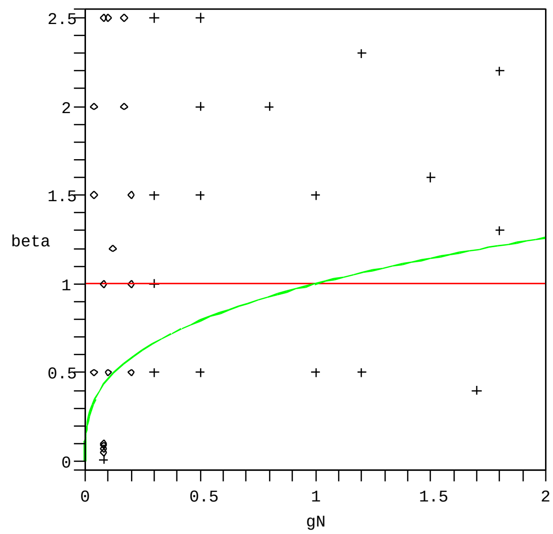

Figure 1: Plot of results for the $g_s N \leq 2$ section of the phase portrait.

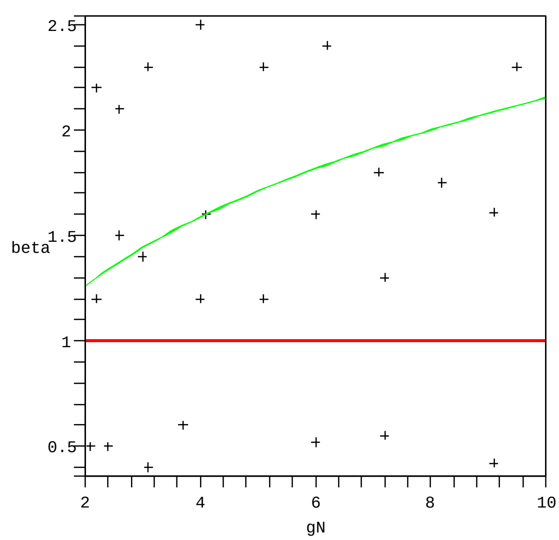

Figure 2: Plot of results for the $g_s N \geq 2$ section of the phase portrait.

terms of the entropy of a free world-volume gauge theory. In the non-conformal cases, using IMSY duality proves to be working, relating strongly coupled gauge theories to the corresponding near-extremal backgrounds. We were able to show that, even after adding general angular momenta quantum numbers, the brane-antibrane model is able to reproduce the supergravity entropy. The exact agreement in the entropy between the microscopic brane-antibrane side and the supergravity side comes at the expense of a renormalization of the mass by a p-dependent power of two, which we interpret as a binding energy, and a renormalization of the angular momenta by a simple factor of two. We also find that the extent of the wavefunction of the $Dp - \overline{Dp}$ system exhibits the same scaling with the mass as does the radius of the horizon of the corresponding black brane.

In the original proposal of [1], the authors argued that the tachyon stabilization at finite temperature and strong coupling could lead to a reappearance of open string degrees of freedom. Even at the field theory level, questions regarding strong coupling dynamics are often hard to answer, but in the special case of D0-$\overline{\text{D0}}$ there is a possibility to address some of the aspects in the context of a simple toy model following the lines of [2, 3]. Using techniques developed there, we found the effective action for the tachyon numerically, in the context of the toy model. We have seen signals of tachyon stabilization in this toy model, at strong open string couplings and low temperatures compared to the string scale. This might open a window to see clearly why it is possible to have a long-lived state of D0-branes and $\overline{\text{D0}}$-branes, without decaying into closed strings at couplings of order one and low temperatures.

Our investigation here was for the case of neutral black branes, where the numbers of microscopic constituent branes N and antibranes $\overline{N}$ are equal. A natural extension of our work here will be to consider the case where the $D = 10$ black branes carry p-brane charge $Q_p = N - \overline{N}$. We hope to report on this story in the future[8].

Inspired by numerical investigations, we might speculate about the possible shape of the potential[9] in this model, say for the simplest case of a diagonal tachyon condensate at finite temperatures

$$\beta F \propto e^{-\gamma |T|^2}(1 + \lambda(N, g_{YM}, \beta)|T|^2 + \ldots) \tag{100}$$

where $\lambda > \gamma$. This potential has one minimum and two symmetrically located maxima. The minimum would correspond to the tachyon stabilization, i.e. the tachyon particle is classically stable[10], even though this vacuum would be unstable due to quantum and thermal fluctuations and the finiteness of the height of the barrier. The tachyon can be expected to decay eventually into closed string radiation. In other words, this temporary and approximate decoupling between the open string modes and the closed strings degrees of freedom – which has apparently manifested itself in our ability to compute the entropy of the closed-string background out of Yang-Mills degrees of freedom in the context of brane-antibrane model – would not last forever, and the tachyon particle ground state would have a finite lifetime. If this picture corresponds well to the true stringy physics, then it would be plausible to think of the unstable

[8] The $p = 3$ case with charge was of course previously considered in [1, 21]. It will be interesting to check whether this agreement holds up also in the *non*-conformal cases. We thank Alberto Guijosa for a discussion in this regard.

[9] at least for $|T|$ in the vicinity on $T = 0$

[10] The kinetic term probably is not canonically normalized in the original field.

maxima discussed above as being the unstable state of a D0-$\overline{\text{D0}}$ pair created from the energy in the gas living on the worldvolume of the clump of D0s and $\overline{\text{D0}}$s, due to the quantum and thermal fluctuations. This interpretation also leads to an estimate for the lifetime. Climbing up the hill to create such a pair requires energies at least of the order of $1/g_s$, so the whole tunnelling process is suppressed by e^{-1/g_s}.

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Appendix

The problem of finding the common solution to a set of nonlinear coupled algebraic equation can be though of a minimization process. To see this, consider the collection of the equations to be solved, $\mathcal{G} = \{f_1 = 0, ..., f_n = 0\}$. Clearly, a quadratic form $\mathcal{H}$ can be made out of this, viz. $\mathcal{H} = \sum_i f_i^2$. If there is a solution to $\mathcal{G}$, this solution would be the global minimum of $\mathcal{H}$ i.e.,$f_1 = 0, ..., f_n = 0$.

To solve this nonlinear system of equations, we use the Monte Carlo local update method ("Metropolis" algorithm). This method is one of the stochastic search methods widely used for minimization of multivariable functions.

This method does not get trapped in local minima, because of a smart choice of control variables. The way this method accomplishes the goal is that, by introducing an unphysical temperature, thermal fluctuations of the configuration vector are allowed to happen in all possible directions in the configuration space - even in the "wrong directions" (which increase the Hamiltonian). By decreasing the temperature very slowly, one lets the vector find the global minimum (or minima). For a vector trapped in a local minimum, there would be a certain probability to climb up the barrier because of the thermal fluctuations, and so the vector could find its way all the way down to the global minimum subject to a large number of iterations. This story is actually very similar to the physics of the annealing process, and it is sometimes called the "stimulated annealing" method. With this method, a vector stores the starting point configuration. This configuration is changed randomly to generate new child configurations. A new configuration gets accepted with the probability 1 if it has a smaller Hamiltonian than its parent, but if it has not then it would be accepted with the following Boltzmann-like probability $P_{\text{acceptance}} \sim e^{-\beta_{\text{fake}}\Delta\mathcal{H}}$, where β_{fake} is a unphysical parameter acting like an inverse temperature, and $\Delta\mathcal{H}$ is the energy difference between the child and parent configurations. After many iterations, and at the same time lowering the temperature slowly, the configuration vector stabilizes on the one(s) which minimize(s) the $\mathcal{H}$.

The Metropolis algorithm has strong advantages over the Newton-Raphson (NR) method, especially when the dimensionality of the configuration space is big. Under such circumstances, the NR method becomes very inefficient, and its starting point

dependence can be prohibitively large. Practically, this makes it very hard to reach the solution in reasonable computer time. The great advantage of the Metropolis method is the fact that it is almost independent of starting point.[11] Another advantage of Metropolis is that it naturally avoids getting stuck in local minima, a problem to which NR often succumbs.

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